Closing Wed, Jan 13: 2.2
Closing Fri, Jan 15: 2.3

2.3 Limit Strategies (Continued)

Entry Task: Find the limits

1. $\lim_{h\to 0}\left[\frac{(5+h)^2-5}{h}\right]$

2. $\lim_{x\to 16}\left[\frac{x-16}{\sqrt{x}-4}\right]$

Recall: (Limit Flow Chart)

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right]$$

1. Try plugging in the value.
   If denominator ≠ 0, done!

2. **If denom = 0 & numerator ≠ 0**,
   the answer is -∞, +∞ or DNE.
   Examine the sign of the output from each side.

3. **If denom = 0 & numerator = 0**,
   Use algebraic methods discussed in class to simplify and cancel until one of them is not zero.

For the den = 0, num = 0 case, here is a summary of the strategies discussed in lecture (we did an example of each):

Strategy 1: Factor/Cancel

Strategy 2: Simplify Fractions

Strategy 3: Expand/Simplify

Strategy 4: Multiply by Conjugate

Strategy 5: Change Variable

Strategy 6: Compare to other functions (Squeeze Thm)

2.5 Continuity

A function, f(x), is **continuous at x = a** if

$$\lim_{x \to a} f(x) = f(a)$$

this implies three things

1. f(a) is defined.
2. $\lim_{x \to a} f(x)$ exists and is finite
3. they are the same!

Casually, we might say a function is continuous at x = a if you can draw the graph across x = a point without picking up your pencil.

Our textbook also defines

Continuous from the left

$$\lim_{x \to a^-} f(x) = f(a)$$

Continuous from the right

$$\lim_{x \to a^+} f(x) = f(a)$$

The “standard” precalculus functions are **continuous everywhere they are defined**:

polynomials → defined everywhere
sin(x), cos(x) → defined everywhere
e^x → defined everywhere
odd roots → defined everywhere
$\tan^{-1}(x)$ → defined everywhere

Rational Functions → denom ≠ 0
Even Roots → under radical ≥ 0
ln(x) → x > 0
tan(x) → not at x = ±kπ/2
$\sin^{-1}(x)$, $\cos^{-1}(x)$ → -1 ≤ x ≤ 1